Time Needed: 140 minutes

Supplies:

- Teacher access to a projector (with sound) for showing a short video in class
- Student access to computers in class (one computer per pair of students)
- Cut-out slips with voting scenarios (Gerrymandering Slips student Handout)
- Exploration activity sheet (Is it Gerrymandering? Student Handout)
- A map handout for each pair of students (link)

See the end of this lesson for mathematics standards correspondence.

Content Objectives: Students will know:

- The definition of the efficiency gap
- How congressional districting is done and how it relates to voter demographics
- What constitutes gerrymandering

Skill Objectives: Students will be able to:

- Discuss ways in which votes can be "wasted" in an election
- Calculate the efficiency gap for a given state, based on a year's election outcomes

Essential Understandings: Students will understand:

- How the efficiency gap situates within various ways of quantifying gerrymandering, such as those which focus on geometry
- That the quantification of real-life phenomena such as gerrymandering inevitably has limitations

Essential Question: Students will explore:

- What does it mean from a mathematical perspective to “waste a vote”?
- How can we use the idea of a wasted vote to understand partisan gerrymandering?
- What are the political consequences of gerrymandering for our states and our country?
- What does "fairness" mean to you in the context of political representation?
- Will my vote someday matter, if these districts stay the same?

Notes for the Teacher

How should we navigate politics in the mathematics classroom? What are the consequences of including or excluding political contexts in mathematics classes? Oftentimes, we as busy educators choose the path of least resistance—avoid the topic altogether—because there are several elements to consider if not hurdle.

People may use mathematics inappropriately or unethically across history; however, one beauty of mathematical reasoning is that, once we agree on starting assumptions, we can use it as an ostensibly neutral tool. A mission for us as math educators is to explore the mathematical concepts and analyze their application in various real-world contexts, leading our students toward grappling with the question: *Now that we know what we know, what should we do?*

That is the best of all possible worlds, but in this fraught political climate, teachers navigate a stormy sea of potential reactions from school and community members. Even if the math *is* neutral, and the exploration *is* objectively conducted, sometimes the mere mention of a topic (voting, elections, voting rights, districts) can raise hackles. We recommend you share this lesson with math colleagues and administrators in department meetings and or professional learning communities. You might also consider co-teaching this lesson with social studies or history colleagues. Some things to discuss before embarking:

- What other preparation should we do, if any, beyond what is recommended with this lesson?
- How will you approach this complex political topic if students or parents react strongly?
- How will you handle other potential reactions within your community?

Activities

Preparing Your Students for Mathematics Study Related to Politics

It is important for this lesson that students come to class having already familiarized themselves with the context of study: U.S. congressional districting and gerrymandering. Before you assign this reading and analysis pre-work, prepare your students for the forthcoming study and discussions with an introduction such as this one:

We will be developing our mathematics skills while exploring one of the real-world political dilemmas of a democracy: gerrymandering. I will give you a quick preview that this is a topic debated enough in this country such that our differences of opinion made it to the Supreme Court in 2017 and was ruled on in 2018. You may have prior notions about gerrymandering, and if so, I ask you to come to this discussion with an open mind ready to learn how math is used in political contexts. You can draw your own conclusions based on mathematical evidence and the research evidence we gather during this process. We are not here to make political arguments one way or

another; we are here instead to see how math is used in political contexts, and then analyze consequences and developments through that lens. Again, you should walk away from this lesson drawing your own conclusions.

Pre-work

The pre-work is key in order for you to participate in high-level discussion and group problem solving, so be sure to come to class prepared!

- Students should read the webpages and video below in the order they are listed. Students should review the questions below, taking notes as needed while engaging with the materials.
    - U.S. Census Bureau description of districting (link)
    - FairVote description of gerrymandering (link)
    - TedEd video explaining the notions of "cracking" and "packing" (3:52; link)
    - Brennan Center for Justice description of the *Gill v. Whitford* case (link)

- Ask students to answer the following questions. You can divide questions among students depending on their readiness level. Tier 1 questions are knowledge and comprehension; Tier 2 are more advanced, requiring more analysis and evaluation. Let students choose which level they want to attempt.
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    - Tier 1: How are congressional district maps decided on? How often do they change, if at all?
    - Tier 1: Explain, in about two-three sentences and your own words, what gerrymandering is. Be sure to paraphrase, showing me that you have translated official definitions into your own understanding.
    - Tier 1: Define "cracking" and "packing."
    - Tier 1: What are key details of the *Gill v. Whitford* case.
    - Tier 2: Why is the Gill v. Whitford case important?
    - Tier 2: What does "fairness" mean to you in the context of political representation?
    - Tier 2: In your view, what might it mean for a vote to be "wasted"?
    - Tier 1 Bonus: extension activity using questions from the TedEd video.
        - Look up the shape of your district in your state and look at the other districts surrounding yours.
        - How wide does your district stretch across your state?
        - Are all of the districts in your state relatively the same shape?
        - How many other districts does your district touch?
        - Does my district look like a salamander?

Introduction (5 minutes)

Begin the lesson by posing the Essential Question to students:

> *What does it mean from a mathematical perspective to waste a vote, and how can we use that idea to understand partisan gerrymandering?*

Explain to students that today they will use their reflections on fairness and what it means to waste a vote to explore how we might use mathematics—and in particular, the *efficiency gap* metric—to capture partisan gerrymandering. Students will then pick a specific state to analyze the extent to which gerrymandering may be at play in its congressional districts, and make an infographic that argues one way or another.

Whole Class Review of Reading (20 minutes)

Ask for students to volunteer their answers to the knowledge and comprehension (Tier 1) reading and analysis questions. Depending on how much review your class needs, spend as much or as little time as needed.

Post these Tier 2 questions and ask students to turn to a partner and briefly share their answers. If you wish, you can ask students to pair up based on the tiers they chose so that students who didn't attempt Tier 2 are working together, and those who did, are.

- What does "fairness" mean to you in the context of political representation?
- In your view, what might it mean for a vote to be "wasted"?

Small-Group Discussions (15 minutes)

Have students form groups of four. Once in their groups, give each group slips with the four questions below on them. Each student should read an individual question, and then take turns sharing out with their group their question, for the group to discuss. Each question facilitator should take the role of making sure others speak first, and should ensure that everyone has a chance to explain their claim. Below are the scenario questions on the slips:

1. Suppose you live in a district that always "goes blue" in the sense that its representatives are almost always Democrats. If you are among the 10% in the district who tend to vote Republican, does your vote "matter"? Why or why not?
2. Suppose you live in a district that always "goes blue" in the sense that its representatives are almost always Democrats. If you are among the 90% in the district who tend to vote Democrat, does your vote "matter"? Why or why not?
3. Suppose you live in a district that is highly competitive and that the 2016 winner was a Republican, with a final vote tally of 51% Republican votes and 49% Democrat votes. What happens to the votes of the Democrats that comprise the 49%?

4. Does a vote tally tell us whether gerrymandering has occurred? For example, does a vote tally of 60% Democrat votes, and 40% Republican votes—or vice versa—suggest that gerrymandering has occurred?

Once groups have had a few minutes to discuss the scenarios, come together as a class. Ask different groups to read and share their thoughts on each of the slips. Below are ideas for what students should take away from the scenarios:

1. If someone is among the 10% in the district who tend to vote Republican, their vote does not contribute to Republican representation. In that strict sense of the word "matter," their vote does not matter.
2. If someone is among the 90% in the district who tend to vote Democrat, their vote does not contribute to Democrat representation, given that without it, Democrats would still win the election. In that strict sense of the word "matter," their vote does not matter.
3. In this district, the 49% have "wasted" their votes in that they did not contribute to Democrat representation.
4. What is important here is that lopsided results are not inherently indicative of gerrymandering; a vote tally for a single district by itself does not provide information about the extent to which gerrymandering has occurred. One needs information on all of the results in the state. Furthermore, intentionality is a part of the definition of gerrymandering, and it is difficult to capture intentionality if you only have post-hoc data.

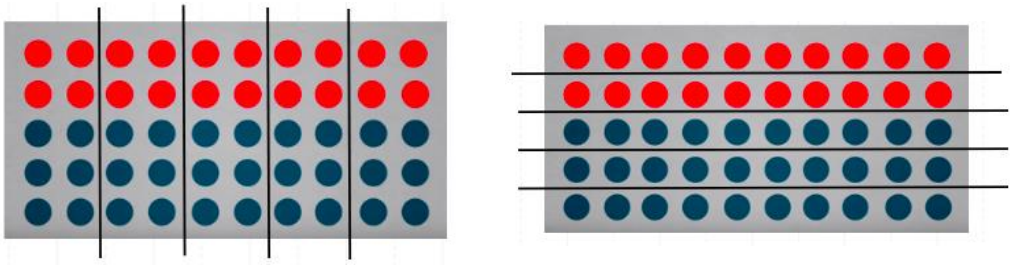
Short Lecture—Introducing the Efficiency Gap (20 minutes)

Having thought through the various scenarios, students will learn about the efficiency gap—a specific tool for measuring the extent to which votes in an election were wasted. Show the short video below to students, asking that they write down the formula for the efficiency gap as it is presented in the video:

WNYC video explaining the efficiency gap (3:00; link)

Think-Pair-Share (10 minutes)

Once the class has watched the video, post the two pictures below, posing the following question in the form of a think-pair-share: Looking at State 1 (image on the left) and State 2 (image on the right), in which state would you expect for the efficiency gap to be higher?



Through the think-pair-share, students should say that the left state would have a higher efficiency gap, given that all of the red votes are wasted. Next, confirm that this is the case by going through an example of actually calculating the efficiency gap with students. Below are two tables with the calculations done.

State 1 (on the left)

District (from left to right)	Red votes	Blue votes	Red wasted votes	Blue wasted votes	Net wasted votes
1	4	6	4	0	4
2	4	6	4	0	4
3	4	6	4	0	4
4	4	6	4	0	4
5	4	6	4	0	4
Total	20	30	20	0	20

Efficiency gap for the first state: (20 – 0)/(50) = 0.4 = 40%

State 2 (on the right)

District (from top to bottom)	Red votes	Blue votes	Red wasted votes	Blue wasted votes	Net wasted votes
1	10	0	4	0	4
2	10	0	4	0	4
3	0	10	0	4	-4
4	0	10	0	4	-4
5	0	10	0	4	-4
Total	20	30	8	12	-4

Efficiency gap for the second state: (8 - 12)/(50) = -0.08 = -8%

What matters is the absolute value of the efficiency gap. Clearly, in the second case, the efficiency gap, at 8% (after taking the absolute value), is much lower than the 40% in the first case.

Pair Exploration Activity: Is it Gerrymandering? (60 minutes)

Explain to students that now they will have the opportunity to explore gerrymandering in one state, the state of North Carolina, as well as in a district of their choosing. Put students into pairs for this exploration activity. Each pair will need a computer and the Is it Gerrymandering? Student Handout to complete their task. Below is the task found on the students' handouts; working through Steps (1) through (6) should take about 60 minutes of class time. Step (7) is for students who finish early.

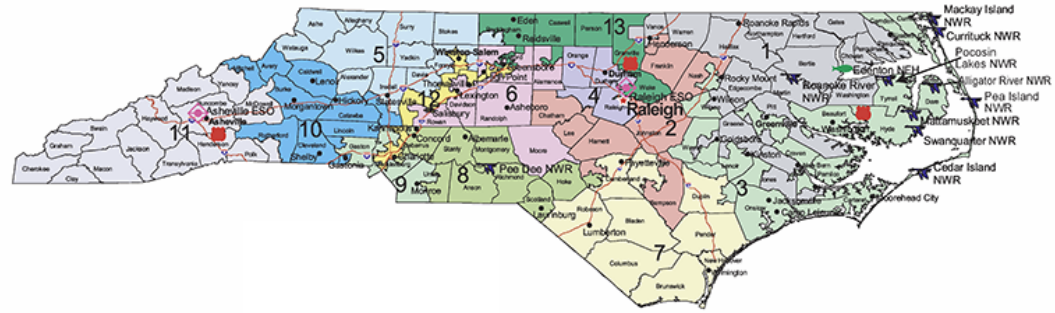
Resources for you to help guide students or to potentially print off for the discussion are linked here:

- This site is helpful for thinking about population density and racial demographics in the U.S.
- See this site for maps of all districts in the U.S.
- Ballotpedia hosts a wealth of information about states and individual districts.
- Many gerrymandering examples come from contexts where Republicans have engaged in gerrymandering. However, it is certainly not the case that they are the only ones who do so. Maryland is a common example of Democrats engaging in the practice; the article linked here explains more.

Student Task (Is it Gerrymandering? handout)

Step 1: Examine the NC congressional districts that were used in its 2012 election. Based on the shapes and what you know about the areas covered (e.g., demographic information about who lives in that district), do you believe gerrymandering occurred? Explain why or why not.

North Carolina Congressional Districts



(Source: https://www.ncpedia.org/congressional-districts)

Step 2: Below is data from the 2012 election in North Carolina. Calculate the efficiency gap using the data.

District	Republican Votes	Democratic Votes
1	76,558	250,948
2	174,565	129,307
3	192,976	112,546
4	91,512	265,432
5	200,083	147,649
6	220,296	141,214
7	167,057	167,590
8	159,226	134,891
9	193,174	170,462
10	189,667	142,822
11	189,289	140,216
12	62,924	246,451
13	225,791	169,637

Step 3: Stephanopoulos and McGhee—the political scientists who created the efficiency gap—suggest that there be a threshold of 8% for determining if a state's district maps are leading to wasted votes. How does North Carolina's efficiency gap compare to that threshold? Based on this, do you have evidence that North Carolina legislators may have drawn unfair maps?

Step 4: If you thought that North Carolina legislators engaged in gerrymandering prior to this election, would you say that they used the method of cracking or packing? How do you know?

Step 5: A question that many people have centers on the extent to which gerrymandering equates to a district looking "funny." You will receive a handout of a U.S. map with congressional districts drawn in. Use this handout to circle—based on your views and those of your partner—what looks to be the “most gerrymandered” district as well as the “least gerrymandered” district. Choose districts in other states in the country. Once you have circled the two districts, visit the site link below to find the state district map of the states your two districts are in; from there, write down the official district names below (e.g., NC District 12 and MI District 9).

https://nationalmap.gov/small_scale/printable/congress.html#list

Step 6: Next, calculate the efficiency gap for each of the two states, based on its 2016 district-level House election results. You will need to create a spreadsheet, or table by hand, that shows how you found the efficiency gap. You can find the data for this task by searching for “2016 House election results in _____,” inserting your state in the blank. For simplicity, only include data from Republican and Democrat voters. With the efficiency gap calculated, is there evidence that legislators—whether intentionally or

not—have drawn district boundaries that lead to a large percentage of wasted votes? Why or why not?

Step 7: If you finish steps (1) through (6) before the rest of the class is ready to move on, visit the following link to explore another means of determining if gerrymandering has occurred: the "compactness" of districts. How does compactness compare to the efficiency gap? What makes the two measures similar or different in what they capture?

As students work through the activity, below are a few questions to push students' thinking as you circulate:

- What makes the shapes of the districts on the map look "funny"? Think about that question in terms of both area and perimeter, for example.
- To what extent can you claim that legislators are acting with intentionality?
- Does going above or beyond the 8% threshold magically result in gerrymandering? What is so special about the 8%, if anything?

Scrumming it Up (20 minutes)

Once students have taken time to complete Steps (1) through (6) of the task above, come together as a class. Students can present their work on each of the different steps. It might make sense to call on specific pairs of students, given their work or method, or you might call on students randomly. In any case, below are ideas to bring out related to each of the steps from the task.

1. District 12 is an infamous example of a district that was drawn to sequester urban voters into a single region. There are several universities in District 12, and university areas tend to lean Democrat; moreover, students should recognize that urban voters generally are more likely to be non-white and to vote Democrat.
2. Cracking is more likely to occur when there is an almost even split in a district between Democrats and Republicans. This appears to have happened in District 9, for example. The cracking in District 9 is facilitated by the packing of voters into District 12. Other answers are possible here; these are just examples.
3. Students should find that the efficiency gap is around 21%, with Democrats having many more of their votes wasted.
4. Because 21% is far above the suggested threshold, we have evidence that—whether intentional or not—Democrats' votes in the state are being wasted, and that the districting is unfairly benefitting Republicans. This is evidence of gerrymandering.
5. Answers will vary. If students do not know where to begin, they might find interesting results in Kentucky, Michigan, Maryland, Texas, and Wisconsin, among other states. If

you are looking for examples where Democrats were responsible for gerrymandering, Maryland is a common state that scholars point to. The article linked here explains more.

6. Answers will vary.
7. Compactness is a geometric measure to try to capture what is happening when we think a district looks "odd." What is problematic about this measure is that odd-looking boundaries could be the result of pre-defined or natural barriers such as state lines and mountains, for example. Another issue is that it only accounts for one district at a time, rather than the state as a whole. These two reasons, among others, make the efficiency gap a more robust measure of gerrymandering.

After having gone over the tasks, explain to students that their individual homework will involve further thinking about gerrymandering in the context of the district that they chose in Step 5.

Homework (90-120 minutes)

There are a number of routes one could take here, especially if you decide to collaborate with a social studies instructor on this topic. Below are questions students could tackle for a short research assignment in relation to this lesson. You might consider converting these prompts into a more extended paper or project.

Directions: Recall the two states you and your partner chose in class. Choose *one* of those states to focus on in this homework assignment, and answer each of the prompts that follow. Below the questions are helpful links to guide your research.

1. What quantitative evidence is there that gerrymandering has (or has not) happened in your state?
2. What do the districts in the state look like, and how is that relevant geographically (population centers, rural areas, voter demographics, etc.)? How are the voting demographics in this area related or connected, and how is that evidence of gerrymandering?
3. What political influences motivated the gerrymandering (or prevented it)? Who (or which party) proposed and approved the district boundaries? Which political party is advantaged or disadvantaged?
4. What are the effects of gerrymandering in your state? Which voters may be under-represented (or over-represented) because of the gerrymandering? What is the election history like in the state?
5. Draw some larger conclusions:
6. What are the political consequences of gerrymandering for our states and our country?
7. Will my vote someday matter, if these districts stay the same?

Mathematics Standards Correlation

Common Core State Standards for Mathematics

Standards for Mathematical Practice
1: Make sense of problems and persevere in solving them
3: Construct viable arguments and critique the reasoning of others
4: Model with mathematics.
5: Use appropriate tools strategically.

Content Standards

Grade 7
Expressions & Equations
B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Geometry
B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Ratios & Proportional Relationships
A.2: Recognize and represent proportional relationships between quantities.

High School
Algebra
A.1: Interpret expressions that represent a quantity in terms of its context.

Geometry
A.1: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Number & Quantity
A.2: Define appropriate quantities for the purpose of descriptive modeling

Statistics & Probability
A.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
B.6: Evaluate reports based on data.