Triangle Midsegments Lesson

Grades: 7th-12th

Time Needed: About 140-150 minutes

Supplies:

- Teacher access to a projector and a laptop with the GeoGebra program (visit website or download the app from iTunes) and Wi-Fi connection to access Google Maps satellite view.
- Student access to a laptop, an iPad or a Smartphone with the GeoGebra app, and Wi-Fi connection to access Google Maps satellite view.
- Duke TIP Triangle Midsegments Handout

See the end of this lesson for mathematics standards correspondence.

Content Objectives: Students will know:

- The definition and the properties of triangle midsegments
- How to use the concepts of ratio and similarity to calculate real-life quantities of objects such as length and distance

Skill Objectives: Students will be able to:

- Identify the characteristics of triangle midsegments
- Use properties of midsegments to solve problems
- Investigate the properties of triangle midsegments using geometry software
- Make conjectures about the properties of shapes, angles, and segments created by triangle midsegments.

Essential Understandings: Students will understand:

- The properties of triangle midsegments and how they are applied to solving skills-based problems and real-life scenarios.
- That the midsegment of a triangle can be used to uncover relationships within a triangle.

Essential Questions: Students will explore:

- How to use coordinate geometry to find relationships within triangles, mainly;
    - to use the midpoint formula to calculate midsegments of triangles,
    - to use the distance formula to examine relationships within triangles.
- How to solve problems that involve measurements of triangles.

Notes for the Teacher

In this lesson, students will be using GeoGebra, a free online interactive Geometry application, to design their own interactive triangle midsegments and construct knowledge by investigating the properties of triangle midsegments in a real-life scenario. They will then use Google Maps satellite view to apply those skills into a real-life scenario. During the activity, they will work in groups, make conjectures, discuss the properties, and solve problems as they answer questions on a handout. In the follow-up activity, they will model/design a solution to a real-life problem scenario and figure out distance of the parking lot using a schools' Google map image and GeoGebra app.

In this activity, I have used the satellite image of my own school. You will find suggestions throughout the lesson on how to personalize it for your own school or building of your choice. Or you can simply choose to use what is provided in this lesson.

This lesson will allow students to apply the properties of triangle midsegments that they just learned using GeoGebra into a real-life, problem-solving situation. They will be given a task—a real-life scenario—in which they will be told that the local elementary school wants to use the middle or high school's parking lot for their carnival, and many children will be disappointed if we can't use the space. Is the space big enough? (Give your students a value based on your own calculations and tell them that the lot should be at least X feet long). Also tell the students that the school's blueprints are missing the length of the parking lot and they are supposed to find a way, using GeoGebra and scientific methods, to accurately calculate that distance. You can add that because this area is off campus or tends to have traffic, it's best to remain inside, so they must find a geometrical calculation method to resolve the situation.

Once they model the school's map in GeoGebra, students will find out that creating a triangle with a midsegment helps them make that calculation, because the midsegment is actually inside campus and they can easily measure that by actually going there and by counting steps and converting to feet. The length of the parking lot will be twice as much per properties of triangle midsegments. GeoGebra app will show them exactly which building is the midsegment, which in my school's case is the distance from the cafeteria to the end of the main building. Everyone can apply this concept to their own school map easily.

The overarching goal in this lesson is to make sure that students understand the reasoning behind the concept of midsegments, how they are constructed, and how they can be applied to different problem solving situations.

The first two skills objectives will help students understand the properties of midsegments and apply them in different problem solving situations while the third one will enable them to construct knowledge using a geometry software; GeoGebra, followed by another activity involving a real-life problem solving situation. The last objective will be implemented throughout the lesson to encourage students to make conjectures about all the shapes, angles, and segments created by triangle midsegments. To achieve this objective you will constantly need to revisit the previous concepts, theorems, and proofs discussed in class regarding midpoint formula, distance formula, parallel and perpendicular lines, transversals, classifying angles and triangles, slopes as well as perimeters and areas of triangles.

This in-depth lesson achieves multiple goals: revisiting a majority of the standards while also allowing students' first-time exposure to the concept of triangle midsegments. Students cans construct meaning by themselves and connect new knowledge and skills to prior skills they have learned in earlier topics such as midpoints, distance formula, slopes, parallel lines, angle pairs, etc. It will provide a meaningful context to put all those earlier standards together and enable students to involve in deep level learning using technology.

This lesson will allow the teacher to incorporate all levels of Bloom's Taxonomy in a single lesson implementation by utilizing technology and a real-life problem-solving scenario through active engagement.

If time permits, students can even be introduced to the concept of fractals (Sierpinski triangle), which is another math standard, and can be asked to design their own Sierpinski fractals using GeoGebra based solely on the concept of triangle midsegments. This could also be used as a new, challenging activity for advanced users who usually finish early.

Teachers who more proficient in using Desmos can use it instead of GeoGebra.

Activities

Preparing Your Students (25-30 minutes)

It is important that students have some basic knowledge of the GeoGebra program. It is a very easy-to-use app for middle and high school level students. I have trained my own students the day before the activity. They were each given an iPad with GeoGebra app already installed. I then demonstrated the tools of the app and explained how each tool can be used to construct different geometrical shapes, create parts (i.e., midsegments, bisectors, segments), and make calculations such as length, distance, area, etc. The app is also accessible on the Web as a web page if app installation is not possible. Let each student practice these skills themselves.

Pre-work

This pre-work has two parts: In the first section, students will need to follow the Midsegments and Problem-Solving handout to design their own triangle midsegments using GeoGebra and to study its properties. In the second section, they will apply the concept into a real-life scenario using a Google satellite image provided by the teacher (see Midsegments and Problem-Solving a handout). If time allows or if you want to incorporate additional technology skills into the lesson, you can ask the students to locate a satellite image, zoom in and crop a section out to use as part of their calculations.

Section 1: Constructing the Midsegment Handout (90 minutes or shorter depending on the groups)

- Start with a brief discussion, asking students, "What do you think a midsegment is?" Connect to a prior skill/vocabulary "midpoint"' to support vocabulary development. Then go over your objectives and standards to encourage interest in the new topic and connect prior skills to the new objectives.
- If you plan on doing the real-world scenario after this exercise, provide a brief preview of the challenge that's ahead and its real-world relevance.
- Ask students to get into groups of three or four and grab an iPad or a laptop. They will follow the GeoGebra instructions on the Midsegments and Problem-Solving handout to design their own triangle midsegments and answer some questions as they move along. It is recommended that each student be asked to complete his or her own handout while working in groups.
- Walk around and ask questions to make sure they are actively engaged in the activity and are building understanding. Ask challenging questions as a formative assessment and also summarize the major findings at the end to make sure everybody understands the properties of midsegments.

The activity follows the following pattern in the same order. Some suggested high-level questioning and feedback techniques will follow.

1. Students start with constructing their own midsegments in GeoGebra (make sure they create the exact same triangle with the correct labels as shown in the template on the handout).
2. They then use coordinate geometry and use the coordinate points of the vertices to calculate the midpoints of two sides of the triangle.
3. Walk around and listen to group discussions. They should be discussing the relationship between the coordinates of the vertices and the method of calculation of the midpoints between them. Ask questions such as
    a. How do you think the coordinates of the vertices are related to finding the midpoints on the sides of the triangle?
    b. Which method/formula you can utilize that we have previously covered in class?

c. How can you compare midpoint to the concept of average? How can you justify your thinking?

The next step is to create the midpoints on the sides and connect them to construct the midsegments. They will answer these questions as a group:

- After you've create your midsegment, what new relationships do you see forming?
- What is your argument about corresponding angles, parallel lines, congruent angles, lengths, slopes, and equations of the line segments? Explain your reasoning.
- Justify your proof(s) after you make claims/conclusions related to the concepts just mentioned above.
- Verify your reasoning as to why those angles are congruent to each other. What evidence makes you think that?
- Compare the slopes of the lines segments and offer a claim regarding the relationship(s) between the slopes of the line segments based on your calculations.

Students will calculate side lengths of various segments using coordinate geometry and deduce relationships and patterns.

- How do you think you can calculate the distance (length) of each side of the triangle?
- Look at the values of the distances (lengths) of each segment you calculated. Do you see any relationships?

Students will write equations for some line segments using coordinate geometry. (This section could be used as an enrichment or differentiation activity for more advanced students if you don't want to use it for all students).

- How is it possible to write the equations of the lines segments?
- What piece of information do you need to infer from the figure?
- How do you justify your reasoning?

Students will now use the GeoGebra app to support their calculations and make conclusions. They will calculate lengths, slopes, and angles using the software and synthesize information to make inferences.

- Based on your calculations using the GeoGebra app, how would you compare and contrast your findings/calculations to your hand calculations?
- Do they support your reasoning? Justify your answers.

- Analyze the similarities and differences. What conjectures can you make regarding the relationship between the specified angles as well as the slopes of both segments?
- What conclusions can you make?
- How would you express your claim(s) in the form of a final statement?

The last step is to create an inside triangle by creating another midpoint on the base and connecting it to both ends of the midsegment of the triangle. They will then answer some questions to make some observations and conjectures regarding the lengths of the parallel segments as well as the perimeters and areas of the big and the small triangles.

- What is happening to the specified segments as you drag them around?
- What relationships do you observe between the pairs of segments? Make arguments and discuss within your groups.
- What conjectures would you come up with regarding the lengths of the given segments?
- What conclusions would you draw?
- Do you think your findings support your conjecture(s)?
- What conjectures would you make regarding the areas and perimeters of the triangles formed?
- Do you see any patterns/relationships?
- How would you justify your answers and make a final claim using the concept of ratios to represent the relationships between their perimeters and areas?

Section 2: Real-life Problem Solving Scenario using GeoGebra and Google Earth (35-45 minutes or shorter depending on the groups).

Next, you can have students do a real-life problem solving activity using Google Earth and GeoGebra. They will gather around the board to answer some questions altogether and discuss ideas to solve the problem using triangle segment concept and technology. This section could be designed as another group-based activity; however, due to time it may take, you may choose to do this part as a whole-group brainstorming session to find a solution to a scenario-based real-life problem together using computer modeling. I will provide insight into both in the sections that follow.

Students will follow the instructions provided in section 2 of the handout to solve the task at hand. I have provided a sample map of my school with instructions on how to create and use it towards the solution of the problem. You may use similar methods to apply the concept to your own school map. If students do not have good technology background with cropping images, zooming in, etc., you are welcome to provide the school's Google Earth image to help them or to shorten the activity.

See images 1 and 2 below to see how I took the school's map and created a triangle with a midsegment using GeoGebra midsegment construction tool, making sure the targeted parking lot becomes the base of the triangle and that another segment (midsegment) remains inside the school campus.

These additions help students easily identify and visit to take measurements using steps and then convert them to feet. Since the base of the triangle (the parking lot) is twice the length of the midsegment of the triangle, all students have to do is to visit where the midsegment falls inside the campus, take measurements, and then multiply it by two to solve the task.

In our case, the midsegment is exactly the distance from our cafeteria to the end of the building and it is about 100-105 steps, which is about 250-262.5 feet based on a man's average stride. Students can easily search for this information online. For women, it will be a few more steps. On average, 1 female stride is about 2.2 feet compared to 2.5 feet for men.

Facilitate the learning process as they argue solution methods within their groups. Ask questions such as:

- How do you think the concept of midsegments could play a role towards a solution method? Justify your reasoning.
- What strategies would you use to verify your reasoning/suggestions/calculations?

Students will then multiply this number by two to find the length of the parking lot, which in our case would be around 500 to 515 feet that is more than what the elementary school would need.

So, it is good news for the elementary school students!

Possible Extensions:

- Instead of length of the parking lot, you can ask for the area of the parking lot, which will require more modeling and calculations, unless you provide the width of the parking lot to your students.
- Advanced students can be asked to use the properties of similar triangles instead of midsegments for a more challenging task and more detailed modeling using GeoGebra.

When this activity is over, you can give students related problems that show up on state level standardized tests, the SAT and the ACT.

Homework

Give students 5 or more problems about triangle midsegments that usually show up in common core based standardized state exams, the SAT and the ACT.

Mathematics Standards Correlation

Common Core State Standards for Mathematics

CCSS.MATH.CONTENT.HSG.CO.C.9

Prove theorems about lines and angles.

CCSS.MATH.CONTENT.HSG.CO.C.10

Prove theorems about triangles.

CCSS.MATH.CONTENT.HSG.CO.D.12

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

CCSS.MATH.CONTENT.HSG.GPE.B.5

Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

CCSS.MATH.CONTENT.HSG.GPE.B.6

Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

CCSS.MATH.CONTENT.HSG.GPE.B.7

Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

CCSS.MATH.CONTENT.HSG.MG.A.1

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

CCSS.MATH.CONTENT.HSG.MG.A.3

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).